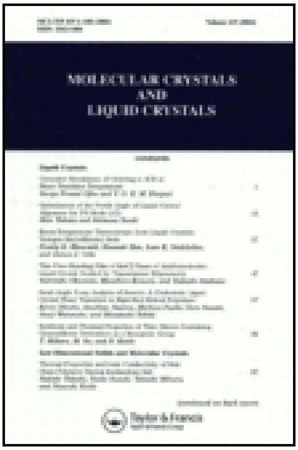

Spiraling and Cyclic Dynamics of Nematicons

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SPIRALING AND CYCLIC DYNAMICS OF NEMATICONS

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Spatial solitons in nematic liquid crystals, or nematicons, can be generated in bulk through a reorientational nonlinearity. We investigate nematicon interactions, showing their behavior related to the inherent nonlocal response of the medium. Solving for both the mechanics and the optics of molecular reorientation in nematic liquid crystals with a nonlinear paraxial version of a Beam Propagator, we demonstrate both in plane and out of plane interactions, disclosing soliton crossing and spiraling.

Keywords: light self-localization; liquid crystals; nematicons; soliton interactions; spatial solitons; spiraling

1. INTRODUCTION

The area of spatial solitons in nonlinear optics has attracted a large interest in the last few decades. Solitons in space possess the fascinating property of overcoming linear diffraction, leading to the formation of self-guided beams, which could be exploited for all-optical information processing, such as signals routing and switching [1,2]. To date, bright spatial solitons have been observed in a wide class of materials, including glass [3,4], semiconductors [5] and photorefractives [6]. With the exception of photorefractives, however, most of them require optical powers of hundreds of Watts in order to observe solitons formation. Nematic liquid crystals (NLC), conversely, represent a valid alternative thanks to a large nonresonant

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nonlinearity (10^9 higher than in Carbon Disulfide): powers of a few mW are sufficient to observe solitons [7,8].

Due to the integrability of the model (represented by the 1-dimensional (1D) nonlinear Schrodinger equation), 1D soliton interactions in Kerr media are fully elastic, i.e., the numbers of solitons is conserved [2,9,10]. On the other hand, materials with a more complex nonlinear response, such as photorefractives and liquid crystals, open the way to the observation of new types of interactions.

Even though – rigorously solitons are stable solutions of nonlinear integrable systems, the term is usually extended to self-localized eigenwaves (solitary waves) of non-integrable sets of equations. In this sense, even in nematic liquid crystals encompassing light-induced molecular reorientation, i.e., a saturable nonlocal response, we refer to solitons (or nematicons) while actually dealing with solitary waves. The peculiar type of nonlinearity leads to inelastic collisions: solitons fusion, crossing and spiraling become possible and nontrivial structures may be generated thru interactions. To date, while photorefractive interactions have been studied both theoretically and experimentally [11–15], only a limited attention has been devoted to nematicons.

In this paper we report on our theoretical results on soliton interactions in nematic liquid crystals. In doing so, because of the underlying physical complexity, we retained the smallest number of approximations and solved for the whole nonlinear model. To this extent, we developed an expressly designed beam propagator (BPM), employing state-of-the-art algorithms [16,17].

2. MODEL

A nematic liquid crystal contains elongated rod-like molecules, which tend to maintain a mean alignment thanks to elastic intermolecular forces. Macroscopically, the medium is birefringent and uniaxial, with $n_\perp$ and $n_{//}$ the ordinary and extraordinary indices of the medium, respectively, while the alignment is usually described by a *director* distribution.

When an optical field acts on NLC, it induces dipoles on the liquid crystal molecules. The torque exerted by the field changes the local orientation of the director. This is the basic mechanism of the optically nonlinear behavior. In suitable geometries (Fig. 1) the field-induced reorientation changes the refractive index experienced by the optical field itself, thus mediating the formation of a waveguide and of a self-localized wave. Let us consider a voltage-biased NLC planar cell as in Figure 1 [8]. Two parallel thin glass plates define the cell and confine the liquid crystal by capillarity. An x

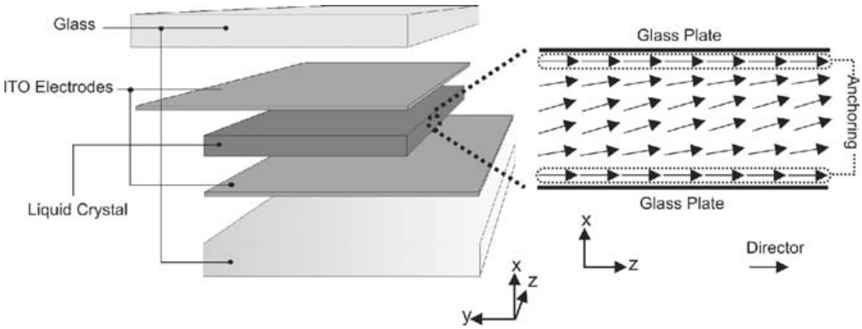

FIGURE 1 Sketch of a voltage-biased nematic liquid crystal cell encompassing a planar anchorage.

polarized light-wave *sees* a refractive index n_e which depends on the director orientation θ:

$$n_e(\theta) = \frac{1}{\sqrt{\frac{\cos^2\theta}{n_\perp^2} + \frac{\sin^2\theta}{n_{//}^2}}} \tag{1}$$

Because of the fluid nature of the NLC, in order to avoid thermal convections it is important to keep the optical amplitude as low as possible, and also eliminate the Freèdericksz threshold [18]. To the latter extent, a static or low-frequency voltage is applied to the NLC by two transparent electrodes of Indium Tin Oxide (ITO), deposited on the glass-NLC interfaces. The applied voltage can be adjusted to presets the molecule orientation to $\theta \cong \theta_0 \neq 0$, thus removing the Freèdericksz transition. An intense light beam propagating inside the NLC and polarized in the extraordinary plane (e.g., along x) can induce changes in the director orientation around θ_0, with an effective nonlinear response maximized around $\theta_0 = \pi/4$ [19].

Let us consider an optical field $E^{opt} = e_x^{opt}\hat{x}\exp(-jk_0 n_e(\theta_0)z)$ propagating inside the bulk NLC, with $k_0 = 2\pi/\lambda$ the vacuum wave-number. To obtain the NLC reorientation equation we used the Frank's energy approach on the spatial director distribution θ [19–21]. To calculate the energy spent by the liquid crystal to maintain a particular director configuration and the field-matter interaction energy, we applied a standard variational method, finding the governing equation for the lowest energy configuration:

$$K \cdot \nabla_\perp^2 \theta + \varepsilon_0 \cdot \left(\frac{E_x^2 \Delta n_{lf}^2}{2} + \frac{\left|e_x^{opt}\right|^2 \Delta n_{hf}^2}{4} \right) \cdot \sin(2\theta) = 0 \tag{2}$$

where E_x is the rms value of the applied static field, K the NLC elastic constant in the single constant approximation (same constant for splay, bend and twist distortions) [19–21], Δn_{lf}^2 and Δn_{hf}^2 the index differences $n_\perp^2 - n_{//}^2$ for quasi-static and high-frequency (optical) fields, respectively.

The evolution equation for the optical field is the TM-like portion of the semi-vectorial formulation of Maxwell's equations [22]. Because of the low refractive index gradient offered by the liquid crystal, we neglect the index derivatives and get:

$$j \cdot 2k_0 n_e(\theta_0)\frac{\partial e_x^{opt}}{\partial z} = \nabla_\perp^2 e_x^{opt} + k_0^2\left(n_e^2(\theta) - n_e^2(\theta_0)\right) \tag{3}$$

where the difference $n_e^2(\theta) - n_e^2(\theta_0)$ represents the nonlinear index perturbation.

3. THE ROLE OF NONLOCALITY

An optically nonlocal response has been investigated in several physical systems for photorefractive and thermal nonlinearities [23–26]. In liquid crystals, the molecular alignment is maintained thanks to intermolecular elastic forces, which give rise to a nonlocal behavior. Changes in refractive index tend to diffuse over a region of larger spatial extent than the perturbative source [2,27]. This is represented in (2) by the transverse laplacian operator, which keeps into account the elastic deformation of the director field. In a nonlocal medium, spatial solitons breath in propagation [27,28] and are stable in two transverse dimensions [29,30]. To understand the role of nonlocality in nematicon interactions, we make use of a simplified version of Eqs. (1) thru (3). Taking a cell much larger than the beam waist, we can use a power series to approximate the nonlinear term in (2). We cast the resulting system in a dimensionless form [28]:

$$\begin{aligned} &j\frac{\partial \mathrm{a}}{\partial \mathrm{z}} + \nabla_\perp^2 \mathrm{a} - \mathrm{a} + \mathrm{a}\phi = 0 \\ &\nabla_\perp^2 \phi - \alpha\phi + \frac{|\mathrm{a}|^2}{2} = 0 \end{aligned} \tag{4}$$

where $\theta = \theta_0 + \Psi$ and $\Psi = (\phi_c/\alpha)\phi(R/R_c\sqrt{\alpha}, z/\alpha Z_c)$ is the angle perturbation of the director, $Z_c = 2k_0 n_e(\theta_0)R_c^2$, $A_\mathrm{c}^2 = 8\Delta \mathrm{n}_\mathrm{lf}^2 \mathrm{E}_\mathrm{x}^4/\pi^2\varepsilon_0 \mathrm{k}_0^2 \Delta\mathrm{n}_\mathrm{hf}^2 \mathrm{K}$, $e_x^{opt} = (A_c/\alpha)a(R/R_c\sqrt{\alpha}, z/\alpha Z_c)\exp(jz/\alpha Z_c)$, $\phi_\mathrm{c} = 2\Delta\mathrm{n}_\mathrm{lf}^2\mathrm{E}_\mathrm{x}^2/\pi \mathrm{k}_0^2\Delta\mathrm{n}_\mathrm{hf}^2\mathrm{K}$, and $\mathrm{R}_\mathrm{c}^2 = \pi\mathrm{K}/2\Delta\mathrm{n}_\mathrm{lf}^2\mathrm{E}_\mathrm{x}^2$. To reveal the physical meaning of the parameter α, we need to express the second of (4) in terms of its Green function. Without loss of generality, we can focus on just one transverse dimension. Performing a Fourier transform in the frequency f, we obtain:

$$\tilde{\phi} = \frac{1}{2}F\{|a|^2\} \cdot \frac{1}{\alpha + k_y^2} \tag{5}$$

Going back to the spatial domain,

$$\phi(y,z) = \frac{1}{4}\int \frac{1}{\sqrt{\alpha}}\exp(-|\xi|/\sqrt{\alpha}) \cdot |a(\xi - y, z)|^2 d\xi \tag{6}$$

The parameter α controls the spatial width of the Green function of (4). When α approaches zero, the Green function becomes wider and the nonlocality of the medium increases. The liquid crystal can then be referred to as a highly nonlocal-medium (HNM), because α is usually small under typical experimental conditions. Let us now consider a set of two solitons (subscripts 1, 2), both in the highly nonlocal regime ($\alpha \rightarrow 0$), launched with an initial phase tilt:

$$a(y,z) = a_1(y,z) \cdot \exp(jk_{y1}y) + a_2(y,z) \cdot \exp(jk_{y2}y) \tag{7}$$

The total intensity $|a|^2$ reads:

$$|a|^2 = |a_1|^2 + |a_2|^2 + 2|a_1| \cdot |a_2| \cos[\Delta k_y y] \tag{8}$$

with $\Delta k_y = k_{y2} - k_{y1}$. The sinusoidal terms in (8) shifts the spectrum of $|a_1||a_2|$ in frequency; in the highly nonlocal case ($\alpha \rightarrow 0$) its contribution lies outside the bandwidth of the transfer function (5). Hence, the nonlocality acts as a low-pass spatial filter, and the resulting medium response is phase independent, i.e., incoherent. Even if the solitons could coherently interact, we expect the nonlocality to give rise to a smoothed-out index perturbation which depends only on the incoherent sum of the soliton amplitudes. Therefore, the role of the nonlocality is to entail incoherent-like soliton interactions [11–13].

Nematicons experience the presence of one another, even if they are launched with large tilt angles or high separation. The nonlocality originates an index distribution (6) wide enough to connect the two localized eigenwaves, even though the field overlap is nearly zero (Fig. 2). The impact of this general behavior is illustrated in Figure 3. Soliton interactions take place whenever an overlap between the index perturbations is present. Hence, two well separated and diverging solitons in a highly nonlocal medium can interact (Fig. 3a–b), while in a local Kerr-like system (Fig. 3c) they propagate independently, because of the localized index perturbation (Fig. 3d).

To proceed further, then, it is useful to distinguish between pairs of nematicons propagating in the same (y, z) plane or in different ones.

4. IN PLANE INTERACTIONS

We investigate nematicon interactions for a variety of input conditions. All simulations are performed numerically solving Eqs. (1)–(3) by the BPM, as

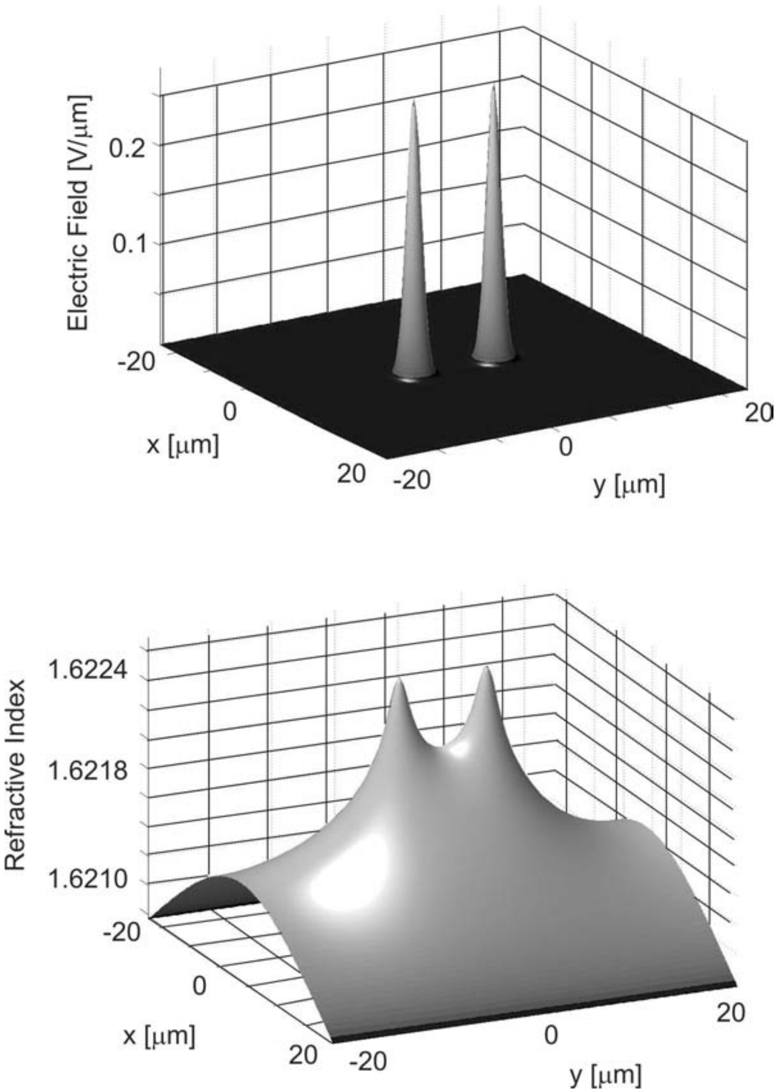

FIGURE 2 (Bottom) Refractive index and (top) electric field distribution after Eqs. (1)–(3) by the BPM. The input optical field is composed by two gaussian beams of waist = 1.5 μm and wavelength 1.064 μm.

described above. Figure 4 shows the propagation of two in-phase identical nematicons, launched parallel to each other. As expected, they cross each other in propagation, attract and undergo a quasi-periodic behavior. This is due to the fact that the forces between two mutually incoherent solitons

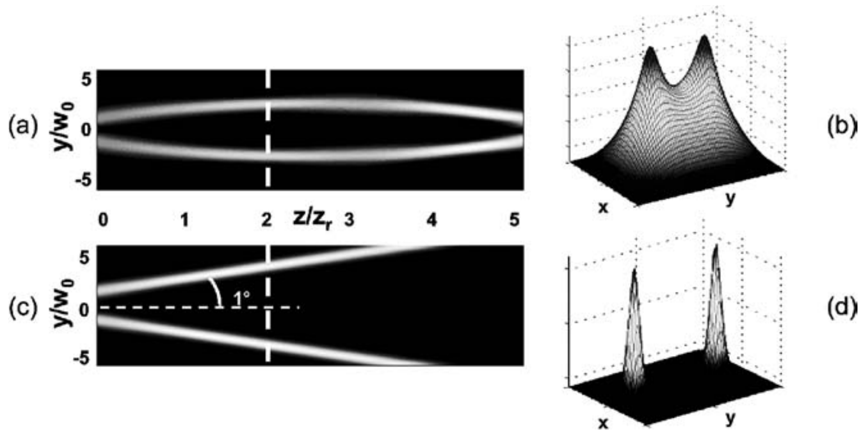

FIGURE 3 Illustration of local and nonlocal soliton interactions. Two (2 + 1)D 10 μm-wide solitons are launched in the y-z plane with an input angle of 2°. In the nonlocal case (a), the solitons interact. The optically-induced index overlap is shown on the top right (b) after 2 diffraction lengths (Z_r). No overlap occurs in a local Kerr medium (d), resulting in independent beam propagation (c).

are always attractive [11,31]. Indeed, one soliton perturbs the other by changing the overall refractive index profile. The second soliton "feels" the waveguide induced by the first, and tends to shift its center of mass

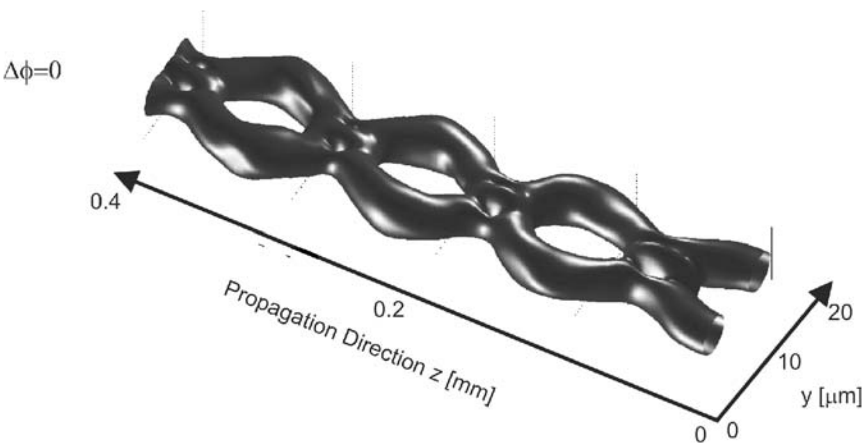

FIGURE 4 Soliton crossing: two identical in-phase nematicons of waist = 2 μm and wavelength 1.55 μm are launched in the (y, z) plane. The beam separation is 9 μm with an input power of 8 mW each.

m_c towards it. Here, m_c is defined as:

$$m_c = \frac{\int r \cdot \left| e_x^{opt} \right|^2 dr}{\int \left| e_x^{opt} \right|^2 dr} \tag{9}$$

with r the transverse radial coordinate, and the integrals extending only over the first soliton. Since light gathers in regions of higher refractive index, each center of mass moves towards the adjacent soliton, resulting into a cyclic crossing and interlacing (Fig. 4). Such behavior has been experimentally verified using the NLC E7, and even exploited to build all-optical logic gates [32]. Moreover, the resulting collision is slightly inelastic and, owing to the finite equivalent mass of each soliton, the

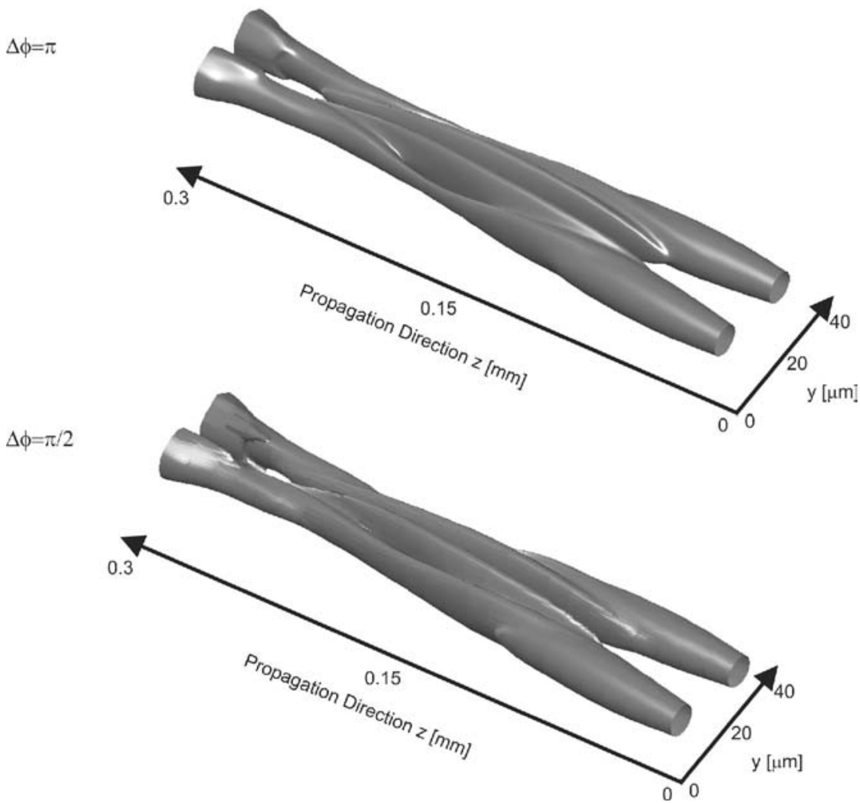

FIGURE 5 Effect of relative phase in nematicon crossing. The input separation is 18 μm while other input conditions are as in Figure 3. The phase change effect is nearly invisible.

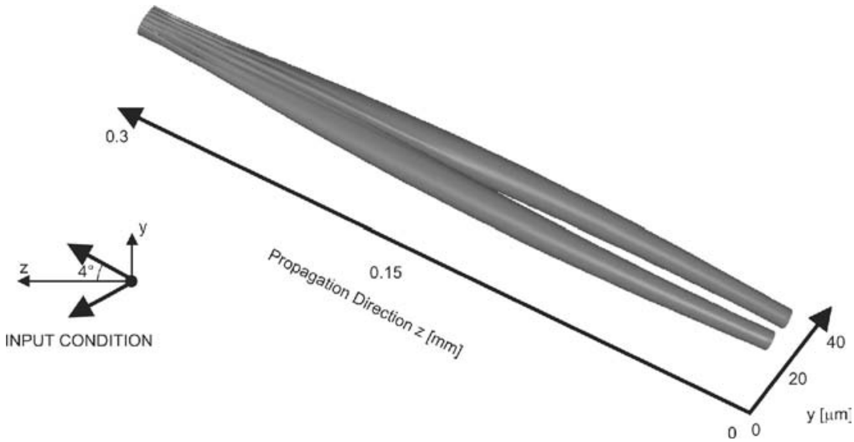

FIGURE 6 Attraction between two diverging nematicons. Two solitons propagate at an initial relative angle of 8° and an input separation of 9 μm. Other input conditions are as in Figure 3.

oscillations become faster and faster with propagation distance, until the solitons eventually fuse [11].

The inherent incoherence of the interaction is aparent in Figure 5, where two solitons are launched in parallel for different input phases. Two overlapping coherent beams yield interference fringes as shown in the figure. Conversely, the refractive perturbation does not follow the field modulation and gives rise to an incoherent interaction.

When two nematicons are launched with an initial phase tilt (Fig. 6), their behavior is substantially the same as in Figure 4: the nonlocal response causes their mutual attraction.

5. OUT OF PLANE INTERACTIONS

One of the most fascinating properties of interacting spatial solitons is their *spiraling* phenomenon. When two solitons carry angular momentum, i.e., they are launched with propagation directions belonging to distinct planes, they gain a mutual repulsive (centrifugal) force [11]. If the attraction compensates the acquired centrifugal force, a spiraling behavior is established, as in Figure 7. Conversely if the launch angles are too small, the momentum cannot balance the attraction and the solitons tend to cross each other and eventually merge, as in the in-plane case. Since incoherent solitons lead to phase independent interactions, the whole phenomenon is governed by the input conditions on relative powers and directions.

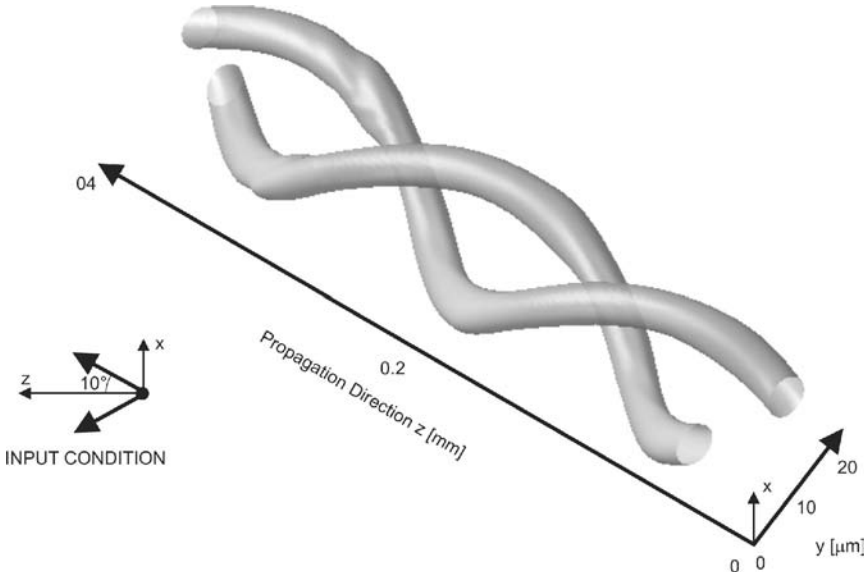

FIGURE 7 Soliton spiraling. Two identical nematicons are launched, with input tilts in the (x, z) plane of 10° and –10°, respectively. Input power is 2 mW each.

6. CONCLUSIONS

We have investigated numerically spatial soliton interactions in nematic liquid crystals. To analyze the role played by the medium nonlocality, we solved the complete nonlinear model taking into account elastic, as well as electrodynamic forces. The relation between a highly nonlocal nonlinearity and the incoherence of interacting solitons has been highlighted, in excellent agreement with previous experimental findings for the in-plane case. We have numerically demonstrated soliton crossing, merging, interlacing and spiraling, phenomena which might pave the way to the realization of novel all-optical devices for switching and readdressing.

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